Implementation of the Non-Linear Gauge into GRACE [1]

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Abstract

A general non-linear gauge condition is implemented into `GRACE`, an automated system for the calculation of physical processes in high-energy physics. This new gauge-fixing is used as a very efficient means to check the results of large scale evaluation in the standard model computed automatically. We report on some systematic test-runs which have been performed for one-loop two-to-two processes to show the validity of the gauge check.

A major part of the theoretical predictions in high-energy physics is based on perturbation theory. However the complexity increases rapidly as one moves to higher orders in perturbations, like when dealing with loop corrections or when dealing with many-body final states. In many instances calculations, if done by hand, become intractable and prone to error. Since perturbation theory is a well-established algorithm, it is possible to construct a system or software to perform these calculations automatically.

There are several systems operating as expert systems for high-energy physics[1]. The *Minami-Tateya* group has developed the system named `GRACE`.[2] Its structure is depicted in the figure. The system can, in principle, deal with the perturbative series up to any order. For instance all diagrams contributing to a process are generated automatically given the order of perturbation, and specifying the particles. However, due to the handling of the loop integrals, practical calculations are, for the moment, restricted to tree and one-loop orders. Two-loop calculation is possible only for some limited cases. The `GRACE` system can work for any type of theory, once the model file is implemented. Here, the model file is a database which stores all component fields and interactions contained in the Lagrangian of the theory. Besides the model file, peripheral parts are sometimes required. For instance, the structure of vertex in the theory is absent in the tool box of `GRACE`, the definition should be added. Also, soft correction factor, kinematics code, the interface to structure functions, parameter control section, and so forth might be supplied if necessary. The system is versatile enough to include new features and such added

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[1]Talk presented by K.Kato at 6-th AIHENP, April 1999, University of Crete.

components would become a part of the new version of GRACE. This report is confined to the calculation based on the so-called standard model. The extension to SUSY is presented in a separated talk.[5]

In contrast to the manual computation, the theoretical prediction from an automated system is obtained by invoking several commands at a terminal with some elapsed time (which is often rather long), due for instance to numerical integrations over phase-space. Especially with an automated system it is difficult to judge the reliability of the final result, hence a need for a built-in automated function to confirm the results. At tree-level, the check of gauge invariance has been shown to be powerful. Within GRACE, the comparison is done between the unitary gauge and the covariant gauge and we observe $\sim 15 (\sim 30)$ digits agreement in double(quadruple) precision computation.
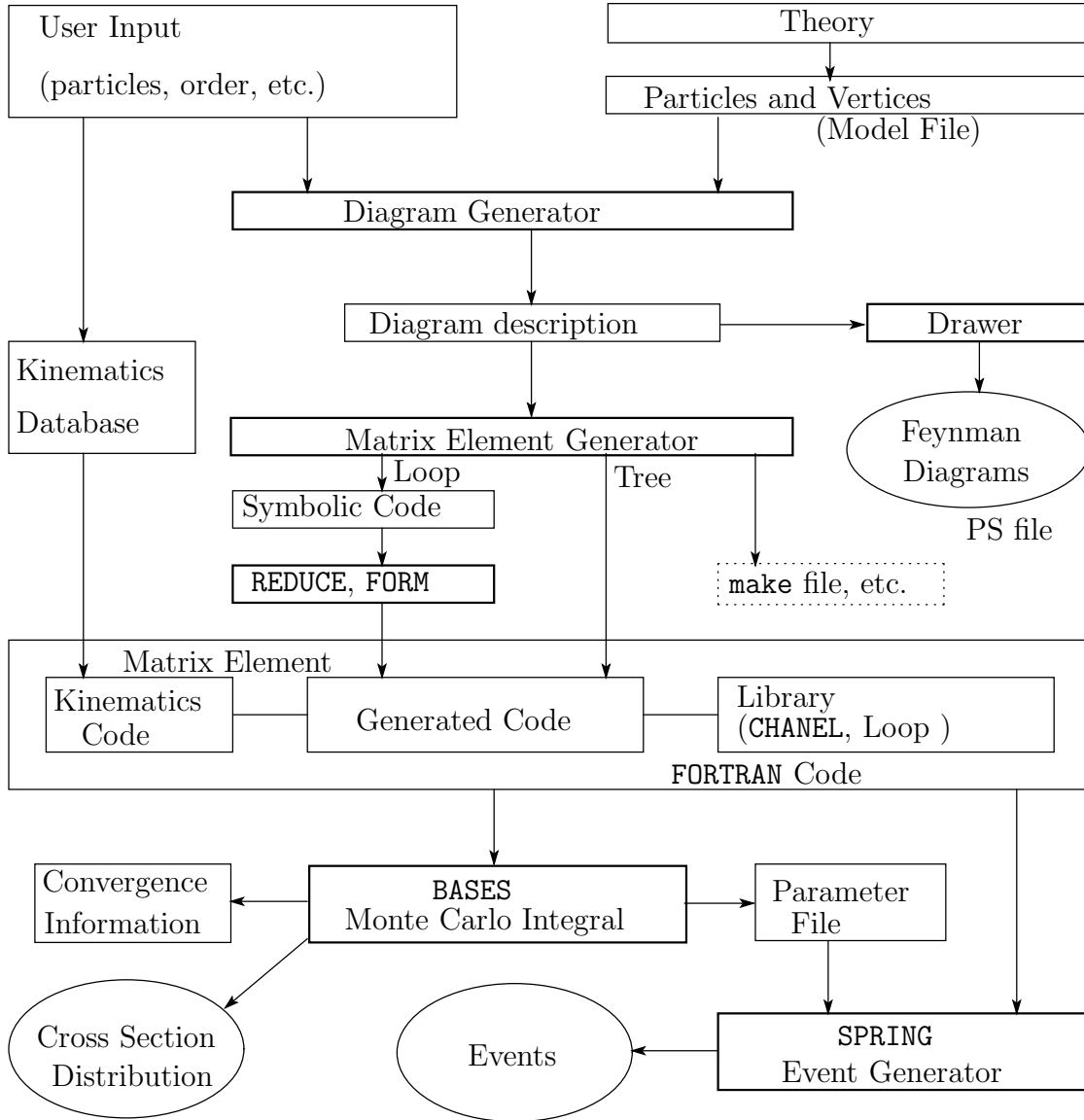
Figure 1: GRACE System Flow

In the one-loop case, the calculation in the unitary gauge does not work well. Within GRACE one of the problems in this gauge is that the library containing the various loop integrals is designed assuming that the numerator of vector particles is $g^{\mu\nu}$. For instance, the library for vertex functions is implemented for the numerator of 3rd order polynomial in the loop momentum, while the handling of 9th order one is required in the unitary gauge. This not only creates very large expressions but also introduces terms with large superficial divergences that eventually need to be canceled precisely between many separate contributions.

The non-linear gauge[3] is introduced to make the gauge-check possible within GRACE. We take a generalized non-linear gauge fixing condition for the standard model[4]:

$$
\mathcal{L}_{\mathrm{GF}} = -\frac{1}{\xi_W} \left| (\partial_\mu - ie\tilde{\alpha}A_\mu - ig\cos\theta_W \tilde{\beta}Z_\mu)W^{+\mu} + \xi_W \frac{g}{2}(v + \tilde{\delta}H + i\tilde{\kappa}\chi_3)\chi^+ \right|^2
$$
$$
-\frac{1}{2\xi_Z}\left(\partial_\mu Z^\mu + \xi_Z \frac{g}{2\cos\theta_W}(v + \tilde{\epsilon}H)\chi_3\right)^2 - \frac{1}{2\xi_A}(\partial_\mu A^\mu)^2
$$

(1)

We take $\xi_W = \xi_Z = \xi_A = 1$ so that the numerator structure of vectors is $g^{\mu\nu}$ as with the usual 'tHooft-Feynman gauge. Any of the other parameters can then provide a gauge check. Then the GRACE library to compute loop amplitudes works without any change. When $\tilde{\alpha} = \tilde{\beta} = \tilde{\delta} = \tilde{\kappa} = \tilde{\epsilon} = 0$, the gauge turns to the 'tHooft-Feynman gauge. For instance, when $\tilde{\alpha} = 1$ ($\tilde{\beta} = -\tan^2\theta_w$), $W^\pm\chi^\mp\gamma$ ($W^\pm\chi^\mp Z$) vertex disappears. Though in the non-linear gauge we have new vertices in the ghost sector, e.g., ghost-ghost-vector-vector vertices, we can reduce the total number of diagrams and therefore speed up the calculation of many processes. Of course the Feynman rules in the model file is revised to take into account the modifications due to the non-linear gauge. This includes all the vertices at tree-level as well as all appropriate counter terms needed for any 1-loop calculation. As a check on the new input file, we have confirmed that the results is unchanged from the original standard model case when all non-linear gauge parameters are 0.

We have tested the system for several tree-level processes as well as one-loop two-to-two processes. Here we report on the tests done at the 1-loop level. The ultraviolet divergence is regularized by dimensional regularization where space-time dimension is $n = 4 - 2\varepsilon$. In the code generated by GRACE, the divergence is kept as a variable Cuv which stands for $1/\varepsilon - \gamma_E + \log 4\pi$. As a first step to check the system, we have compared, for a given random set of the gauge parameters, the results with letting Cuv$= 0$ and that with Cuv$\neq 0$. Since the agreement is exact, the system passes the first diagnosis, renormalizability. Then, we proceed to check that the finite result is independent of the choice of set for the non-linear gauge parameters.

Some of the processes are listed in the table below. The center-of-mass energy and the masses used in the computation are as follows: $W = 500\mathrm{GeV}, M_Z = 91.187\mathrm{GeV}, M_W = 80.37\mathrm{GeV}, M_H = 100\mathrm{GeV}, M_t = 174\mathrm{GeV}$. For the regularization of the infra-red divergences, we introduce a fictitious photon mass, and its value in the computation is $\lambda = 10^{-6}\mathrm{GeV}$. In the table, we present the value of

$$
\left(\frac{d\sigma^{1-loop}}{d\cos\theta} - \frac{d\sigma^{tree}}{d\cos\theta}\right)_{\theta=10°(\cos\theta=0.985)} \quad \propto \quad 2\Re\left(T^{loop}\cdot T^{tree\ \dagger}\right),
$$

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and LG stands for the linear gauge('tHooft-Feynman gauge) case with `Cuv=0` and NLG does for the non-linear gauge case with `Cuv=1`, $\tilde{\alpha} = \tilde{\beta} = 1$, $\tilde{\delta} = \tilde{\kappa} = \tilde{\epsilon} = 0$. The latter case corresponds to the background gauge formalism.[6] The number of diagrams depends on the choice of the gauge-fixing condition since some vertices are not present in some gauges. The counting of the total number of diagrams in the Table refers to all possible diagrams and therefore with appropriate choices of gauge this number may be reduced. The number of diagrams involving counter-terms insertions and that with self-energy contributions is denoted by CT and SE, respectively.

process	Number of Diagrams			$d\sigma/d\cos\theta$[pb]	
	tree	1-loop	(CT:SE)	LG	NLG
$e^+e^- \to t\bar{t}$	2	52	$(4:6)$	-0.870876519	-0.87087619
$e^+e^- \to HZ$	1	119	$(3:4)$	-0.03174046785	-0.03174046785
$e^+e^- \to W^-W^+$	3	152	$(5:5)$	-0.9963368092	-0.9964451605
$t\bar{b} \to t\bar{b}$	6	268	$(12:14)$	33.75029132	33.75132514
$W^+W^- \to t\bar{t}$	4	238	$(8:6)$	-0.08938607492	-0.08938607286
$ZH \to t\bar{t}$	4	352	$(8:8)$	2.672194263	2.672194265
$W^+\gamma \to t\bar{b}$	4	238	$(8:6)$	-0.5998664910	-0.5998663896
$W^+Z \to t\bar{b}$	4	282	$(8:6)$	-0.2216982981	-0.2216940817
$W^+H \to t\bar{b}$	4	283	$(8:6)$	0.04785521591	0.0478570236

We note that the agreement and hence the gauge independence of the result is excellent. The accuracy is only limited by that of the evaluation of the numerical loop integrals. Through detailed inspections of individual diagrams we have noted that, expectedly, the gauge independence requires different diagrams to combine in order to produce a gauge-parameter independent result. In this sense, the check by gauge invariance is a powerful diagnostics. The comparison is still in progress for other processes at one-loop to establish the validity of the system. We can then proceed to more complicated processes, *i.e.*, one-loop two-to-three or two-to-four processes and can confirm the results of large scale computation by the non-linear gauge method presented here.

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